Formulas for Math 9B

Integral Calculus

APEX CALCULUS

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Index of Tables of Old Formulas:

- Differentiation Rules
- Trigonometry
- Areas & Volumes
- Basic Algebra

Index of New Formulas:

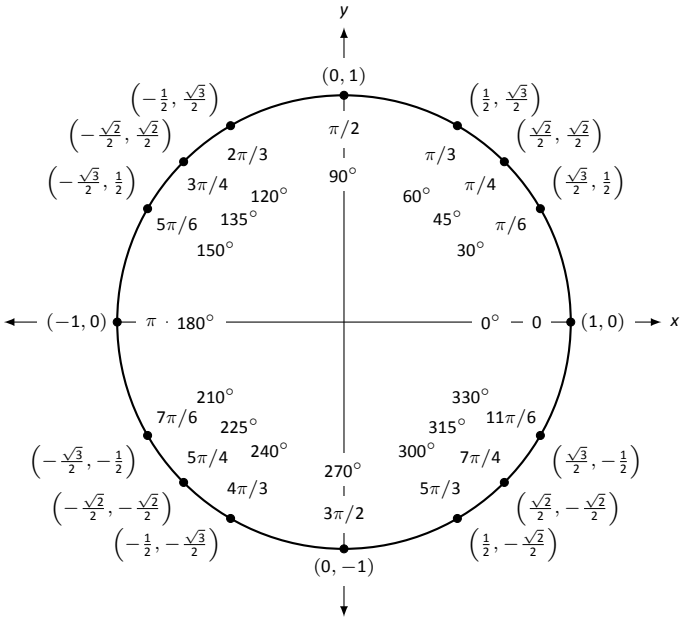
- Integration Rules
- Other Formulas: Summations, Fundamental Theorem, Arc Length, Area Between Curves, Area by Cross-Sections, Disks, Washers, Shells, Solids of Revolution.

Differentiation Rules (Math 9A)

1. $\frac{d}{dx}(cx) = c$
2. $\frac{d}{dx}(u \pm v) = u' \pm v'$
3. $\frac{d}{dx}(u \cdot v) = uv' + u'v$
4. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}(u(v)) = u'(v)v'$
6. $\frac{d}{dx}(c) = 0$
7. $\frac{d}{dx}(x) = 1$
8. $\frac{d}{dx}(x^n) = nx^{n-1}$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$
11. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$
13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
16. $\frac{d}{dx}(\sec x) = \sec x \tan x$
17. $\frac{d}{dx}(\tan x) = \sec^2 x$
18. $\frac{d}{dx}(\cot x) = -\csc^2 x$
19. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$
22. $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$
23. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$
24. $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$
25. $\frac{d}{dx}(\cosh x) = \sinh x$
26. $\frac{d}{dx}(\sinh x) = \cosh x$
27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
28. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
29. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$
31. $\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}}$
32. $\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}}$
33. $\frac{d}{dx}\left(\operatorname{sech}^{-1} x\right) = \frac{-1}{x\sqrt{1-x^2}}$
34. $\frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = \frac{-1}{|x|\sqrt{1+x^2}}$
35. $\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$
36. $\frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}$

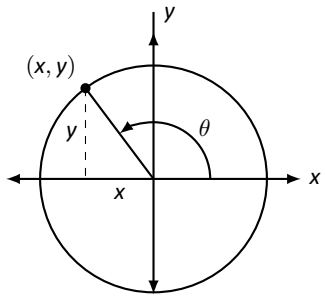
Trigonometry

The Unit Circle



Definitions of the Trigonometric Functions

Unit Circle Definition

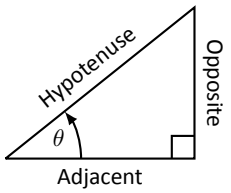


$$\sin\theta = y \qquad \cos\theta = x$$

$$\csc\theta = \frac{1}{y} \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Right Triangle Definition



$$\sin\theta = \frac{\text{O}}{\text{H}} \qquad \csc\theta = \frac{\text{H}}{\text{O}}$$

$$\cos\theta = \frac{\text{A}}{\text{H}} \qquad \sec\theta = \frac{\text{H}}{\text{A}}$$

$$\tan\theta = \frac{\text{O}}{\text{A}} \qquad \cot\theta = \frac{\text{A}}{\text{O}}$$

Common Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Power–Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Double Angle Formulas

$$\sin 2x = 2\sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Other Trigonometric Identities

Angle Sum/Difference Formulas

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Even/Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

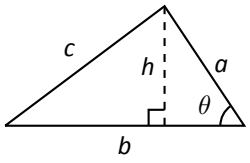
Areas and Volumes

Triangles

$h = a \sin\theta$

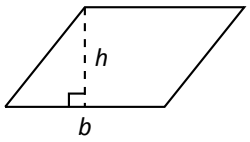
Area = $\frac{1}{2}bh$

Law of Cosines:
$c^2 = a^2 + b^2 - 2ab\cos\theta$



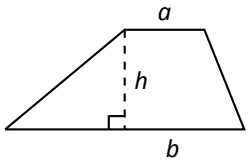
Parallelograms

Area = bh



Trapezoids

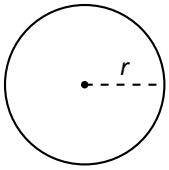
Area = $\frac{1}{2}(a+b)h$



Circles

Area = πr^2

Circumference = $2\pi r$

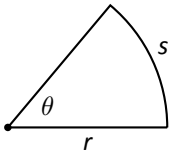


Sectors of Circles

θ in radians

Area = $\frac{1}{2}\theta r^2$

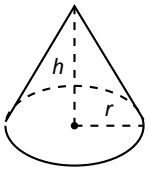
$s = r\theta$



Right Circular Cone

Volume = $\frac{1}{3}\pi r^2 h$

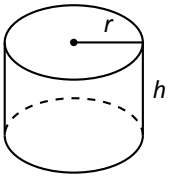
Surface Area =
$\pi r\sqrt{r^2+h^2} + \pi r^2$



Right Circular Cylinder

Volume = $\pi r^2 h$

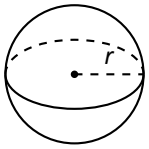
Surface Area =
$2\pi rh + 2\pi r^2$



Sphere

Volume = $\frac{4}{3}\pi r^3$

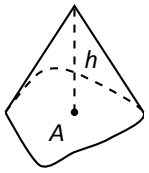
Surface Area = $4\pi r^2$



General Cone

Area of Base = A

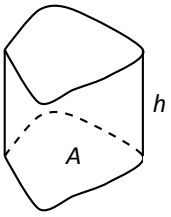
Volume = $\frac{1}{3}Ah$



General Right Cylinder

Area of Base = A

Volume = Ah



Basic Algebra

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \le b^2 - 4ac$, then the real zeros of p are $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Special Factors

$x^2 - a^2 = (x - a)(x + a)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$

$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

Factoring by Grouping

$acx^3 + adx^2 + bcx + bd = ax^2(cs + d) + b(cx + d) = (ax^2 + b)(cx + d)$

Arithmetic Operations

$ab + ac = a(b + c)$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

$a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{a - b}{c - d} = \frac{b - a}{d - c}$

$\frac{ab + ac}{a} = b + c$

Exponents and Radicals

$a^0 = 1, \quad a \neq 0$

$(ab)^x = a^x b^x$

$a^x a^y = a^{x+y}$

$\sqrt{a} = a^{1/2}$

$\frac{a^x}{a^y} = a^{x-y}$

$\sqrt[n]{a} = a^{1/n}$

$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$\sqrt[n]{a^m} = a^{m/n}$

$a^{-x} = \frac{1}{a^x}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$(a^x)^y = a^{xy}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

New Formulas Math 9B

APEX CALCULUS

Integral Calculus

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Differentiation Rules (From Math 9A - Written here to compare with integration rules below)

1. $\frac{d}{dx}(cx) = c$
2. $\frac{d}{dx}(u \pm v) = u' \pm v'$
3. $\frac{d}{dx}(u \cdot v) = uv' + u'v$
4. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}(u(v)) = u'(v)v'$
6. $\frac{d}{dx}(c) = 0$
7. $\frac{d}{dx}(x) = 1$
8. $\frac{d}{dx}(x^n) = nx^{n-1}$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$
11. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$
13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
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17. $\frac{d}{dx}(\tan x) = \sec^2 x$
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19. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$
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21. $\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$
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23. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$
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25. $\frac{d}{dx}(\cosh x) = \sinh x$
26. $\frac{d}{dx}(\sinh x) = \cosh x$
27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
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29. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
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31. $\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}}$
32. $\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}}$
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34. $\frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = \frac{-1}{|x|\sqrt{1+x^2}}$
35. $\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$
36. $\frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}$

Integration Rules (New to Math 9B)

1. $\int c \cdot f(x)\,dx = c\int f(x)\,dx$
2. $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int 0\,dx = C$
4. $\int 1\,dx = x + C$
5. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C,\ n \neq -1$ $n \neq -1$
6. $\int e^x\,dx = e^x + C$
7. $\int a^x\,dx = \frac{1}{\ln a} \cdot a^x + C$
8. $\int \frac{1}{x}\,dx = \ln|x| + C$
9. $\int \cos x\,dx = \sin x + C$
10. $\int \sin x\,dx = -\cos x + C$
11. $\int \tan x\,dx = -\ln|\cos x| + C$
12. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
13. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
14. $\int \cot x\,dx = \ln|\sin x| + C$
15. $\int \sec^2 x\,dx = \tan x + C$
16. $\int \csc^2 x\,dx = -\cot x + C$
17. $\int \sec x \tan x\,dx = \sec x + C$
18. $\int \csc x \cot x\,dx = -\csc x + C$
19. $\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$
20. $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
21. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
22. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
23. $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right) + C$
24. $\int \cosh x\,dx = \sinh x + C$
25. $\int \sinh x\,dx = \cosh x + C$
26. $\int \tanh x\,dx = \ln(\cosh x) + C$
27. $\int \coth x\,dx = \ln|\sinh x| + C$
28. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x + \sqrt{x^2-a^2}\right| + C$
29. $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|x + \sqrt{x^2+a^2}\right| + C$
30. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2}\ln\left|\frac{a+x}{a-x}\right| + C$
31. $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \frac{1}{a}\ln\left(\frac{x}{a+\sqrt{a^2-x^2}}\right) + C$
32. $\int \frac{1}{x\sqrt{x^2+a^2}}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$

Formulas New To Math 9B:

Summation Formulas:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Definite integral is Limit of Riemann Sums:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \left[f(c_1) + f(c_2) + f(c_3) + f(c_4) + \ldots + f(c_{n\text{-}1}) + f(c_n) \right]$$

Example: Left-hand Riemann Sum $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \left[f(x_1) + f(x_2) + f(x_3) + \ldots + f(x_{n\text{-}1}) + f(x_n) \right]$

Example: RIght-hand Riemann Sum $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \left[f(x_2) + f(x_3) + f(x_4) + \ldots + f(x_n) + f(x_{n+1}) \right]$

FUNDAMENTAL THEOREM OF CALCULUS: Part 1 and Part 2

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt \right) = f(x) \qquad \int_a^b F'(x)\,dx = F(b) - F(a)$$

Area Between Curves:

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx$$

Volume By Cross-Sectional Area:

$$V = \int_a^b A(x)\,dx$$

Arc Length:

$$L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$

	Disk Method	**Washer Method**	**Shell Method**
Horizontal Axis	$\pi \int_a^b R(x)^2\,dx$	$\pi \int_a^b \big(R(x)^2 - r(x)^2\big)\,dx$	$2\pi \int_c^d r(y)h(y)\,dy$
Vertical Axis	$\pi \int_c^d R(y)^2\,dy$	$\pi \int_c^d \big(R(y)^2 - r(y)^2\big)\,dy$	$2\pi \int_a^b r(x)h(x)\,dx$

Surfaces of Revolution with Respect to *x*-axis and *y*-axis:

$$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\,dx$$
(where $f(x) \geq 0$)

$$S = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\,dx$$
(where $a, b \geq 0$)